JAIRO CHARRIS SEMINAR 2010

Algebraic Aspects of Darboux Transformations, Quantum Integrable Systems and Supersymmetric Quantum Mechanics

http://ima.usergioarboleda.edu.co/SJCH/JCHS2010.htm

Scientific program

Download **here** the pdf version

Schedule

Time	Wednesday – Chairman:	Thursday – Chairman:	Friday	Saturday – Chairman:
8h30	Plenary Talk: V. Spiridonov	Plenary Talk: V. Sokolov	Social meeting- Visit to Tayrona Natural Park	Plenary Talk: V. Ovsienko
10h00	Coffee-Poster 1	Coffee-Poster 2		Coffee-Poster 3
10h30	Talk: P. Roy	Talk: W. L Yang		Talk: J. M. Guilarte
11h30	Talk: J.J. Morales-Ruiz	Talk: H. Falomir		Talk: E. Reyes
12h30	Lunch	Lunch		Lunch
14h	Plenary Talk: Y. Brezhnev	Plenary Talk: R. Milson		Cultural event
15h30	Coffee-Poster1	Coffee-Poster2		Coffee-Poster 3
16h	Talk: L. Garza	Talk: V. Hussin		
After 19h	Presentation of IMA	Visit to Santa Marta Historical Center		

Abstracts

Conferences

Speaker: Yurii Brezhnev, (Kaliningrad State University, Russia)

Title: Exact solvable soliton and algebro-geometric potentials in the light of differential Galois (Picard-Vessiot) theory. Why and where do the theta-functions come from?

Abstract: *We give a treatment of exact solvable potentials for spectral problems defined by ODE's in the language of a theory which begins from the strict definition of how the integrability should be perceived. Complete and correct answer is given by the theory which is known presently as differential Galois one or as the Picard-Vessiot theory. All the famous soliton and algebro-geometric potentials fit this theory as the cases having solutions in exact quadratures without(!) resorting to any specfunctions. The well-known approaches with use of θ-series flow entirely from that view on the problem and this point of view trivializes reading of the theta-function technique. Moreover we show that theta-functions themselves -- in the case of g=1, i.e. Jacobi's θ-functions – admit a complete `integrable description' in the sense that they possess all the attributes of exact solvable models: Hamiltonians and Poisson structures for algebraically integrable ODE's. In this regard the θ-functions are not to be considered as special functions.*

Speaker: Horacio Falomir (U. Nacional de la Plata, Argentina)

Title: Self-adjoint extensions and SUSY breaking in a model of supersymmetric Quantum Mechanics

Abstract: *We consider the self-adjoint extensions (SAE) of the symmetric supercharges and Hamiltonian for a model of SUSY Quantum Mechanics in the half line with a singular superpotential. We show that only for two particular SAE, whose domains are scale invariant, the algebra of N=2 SUSY is realized, one with manifest SUSY and the other with spontaneously broken SUSY. Otherwise, only the N=1 SUSY algebra is obtained, with spontaneously broken SUSY and non degenerate energy spectrum.*

Speaker: Luis Garza Gaona (Universidad Autónoma de Tamaulipas,México)

Title: Spectral transformations of orthogonal polynomials: A connection to integrable systems.

Abstract: *Orthogonal polynomials with respect to measures supported on the real line satisfy a three term recurrence relation that can be represented on matrix form by means of a symmetric tri-diagonal matrix known as Jacobi matrix. When certain spectral transformations are applied to the orthogonality measure, it is possible to obtain the Jacobi matrix associated to the perturbed measure using LU and UL factorizations of the original Jacobi matrix. These factorizations are also known in the literature as Darboux transformations. Analogously, spectral transformations to measures supported on the unit circle can be expressed by means of (almost) QR factorizations of a Hessenberg matrix. In this contribution, we present some results in this direction and their relation with some integrable systems when a temporal variable is introduced on the entries of such matrices. Some interesting open problems will be briefly discussed.*

*Joint work with Francisco Marcellán (Universidad Carlos III de Madrid)

Key words: Spectral transformations, Orthogonal polynomials, Jacobi matrices, Darboux transformations, Integrable systems.

2000 AMS classification: 42C05, 15A23.

Speaker: Véronique Hussin (Montreal University, Canada)

Title: Two-dimensional Morse potentials and quantum coherent and squeezed state

Abstract: *For one dimensional non relativistic quantum systems, coherent and squeezed states have been largely studied, starting from those of the harmonic oscillator. They have been used in order to understand the quantum properties of light. They have also been generalized in different ways using methods of group and supergroup theories. In particular, we have recently been able to construct different types of those states for the completely solvable system of a one-dimensional anharmonic oscillator. Such a system represents well the anharmonic vibrations in diatomic molecules and the potential*

associated is the well-known Morse potential. Properties of those states in terms of squeezing, minimum Heisenberg uncertainty principle, space localization and statistics have been studied.

Solvable higher dimensional quantum systems have been classified using different approaches like algebraic, factorization, shape invariance and also supersymmetric methods. In particular, the spectra of partner Hamiltonians for the Morse potential in two dimensions have been studied using supersymmetry and second order supercharges. We thus have extended the construction of coherent and squeezed states to these systems. Since, such systems present degeneracies in their spectra, those states must be carefully defined. As in the one dimensional case, good spatial localization of those states are obtained by adjusting the coherent and squeezing parameters.

Speaker: Juan Mateos Guilarte (U. Salamanca, Spain)

Title: Supersymmetric Quantum Mechanics based on Classical Integrable Systems

Abstract: *Starting from the Kepler (one Newtonian center, in various dimensions and topologies) and the Euler (two planar Newtonian centers) problems we will describe how Supersymmetric Quantum Mechanical systems can be constructed. In the SUSY Kepler problem the spectrum can be obtained by algebraic means with the help of the Runge-Lenz integral of motion. In the other case, the SUSY two center problem, the spectrum is determined from the QES Razavy and Whittaker-Hill equations.*

Speaker: Robert Milson (Dalhousie University, Canada)

Title: Exceptional orthogonal polynomials and the Darboux transformation.

Abstract: *We adapt the notion of the Darboux transformation to the context of polynomial Sturm-Liouville problems. As an application, we characterize the recently described exceptional polynomial families in terms of an isospectral Darboux fransformation. The latter families are characterized as complete orthogonal polynomial systems $p_m, p_{m+1}, \ldots $ that are defined by a second order eigenvalue equations, and where $\deg p_j = j$ and $m>0$. Like the classical families, these orthogonal polynomials admit raising and lowering operators. We show that this shape-*

invariance property is a direct consequence of the permutability property of the Darboux-Crum transformation.

Speaker: Juan J. Morales-Ruiz (Universidad Politecnica de Madrid,Spain)

Title: A Galoisian Approach to Supersymmetric Quantum Mechanics

Abstract: *In this work we will show that the Galois Theory of Linear Differential Equations, also called the Picard-Vessiot Theory, is the natural framework to study the solvability in closed form of the Schrodinger equation of Quantum Mechanics. Thus, all the cases where this equation was solved by quadratures can be classiffied using the Picard-Vessiot Theory. As a by product, we obtain a Galoisian interpretation of the Supersymmetric Quantum Mechanics. This is a joint work with Jacques-Arthur Weil and Primitivo B. Acosta-Humanez. A preliminary version of this work can be found at http://arxiv.org/abs/0906.3532.*

Speaker: Valentin Ovsienko (Universite Claude Bernard Lyon 1, France)

Title: The Pentagram map: a discrete integrable system

Abstract: *The pentagram map is a projectively natural iteration defined on polygons. We find a Poisson structure on the space of twisted polygons and show that the pentagram map relative to this Poisson structure is discrete completely integrable system. We also explain how the pentagram map, in the continuous limit, corresponds to the classical Boussinesq equation. The Poisson structure we attach to the pentagram map is a discrete version of the first Poisson structure associated with the Boussinesq equation.*

Speaker: Enrique Reyes (Universidad Santiago de Chile)

Title: Integrability properties of the Camassa-Holm equation and a Darboux transformation

Abstract: *In this talk I will review some geometric properties of the important Camassa-Holm equation*

2\,u_{{x}}\,u_{{xx}} + u\,u_{{xxx}} = u_{{t}} - u_{{xxt}} +

3\,u_{{x}}\,u \; \label{ch}

introduced by R. Camassa and D. Holm in 1993 as model for shallow water waves of moderate amplitude. In particular, I will show that:

- *It is the integrability condition of an over-determined $sl(2,{\bf R})$-valued linear problem; it admits soliton solutions and it can be solved via scattering/inverse scattering.*
- *It admits a quadratic pseudo-potential;*
- *It possesses an infinite number of {\em local} conservation laws;*
- *It admits a Lie algebra of {\em nonlocal} symmetries.*

As an application, I will present a recent explicit construction of a Darboux transformation for the Camassa-Holm equation.

There also exists an "associated Camassa-Holm" (ACH) equation constructed via hodographic transformations. All the properties above can be "translated" into properties of ACH.

References:

E.G. Reyes, Geometric integrability of the Camassa--Holm equation. {Letters in Mathematical Physics} 59, no. 2 (2002), 117--131.

R. Hernández-Heredero and E.G. Reyes, Nonlocal symmetries and a Darboux transformation for the Camassa-Holm equation. {J. Physics A: Mathematical and Theoretical}, Fast Track Communication, April 2009.

R. Hernández-Heredero and E.G. Reyes, Geometric integrability of the Camassa-Holm equation. II. Submitted, 2010.

Speaker: Pinaki Roy (Indian Statistical Institute, India)

Title: Conditionally exactly solvable potentials and exceptional orthogonal polynomials

Abstract: *We study the solutions of certain potentials associated with broken as well unbroken supersymmetry. In particular it is shown that these solutions belong to the category of Exceptional Orthogonal Polynomials. We have studied various properties of these polynomials such as Rodrigues formula, Recurrence relation, Generating function etc. This talk is based on our recent paper published with the same title in J. Math. Phys. 51, 042101 (2010); doi:10.1063/1.3339676.*

Speaker: Vladimir Sokolov, Landau Institute, Russia

Title: Compatible associative algebras and integrable matrix ODEs.

Abstract: *Given an associative multiplication in the matrix algebra compatible with the usual one or, in other words, a linear associative deformation of the matrix algebra, we construct a solution to the classical Yang-Baxter equation. We also develop a theory of such deformations and construct numerous examples. It turns out that these deformations are in one-to-one correspondence with representations of certain algebraic structures, which we call M-structures. We describe an important class of M-structures. The classification of these M-structures naturally leads to affine Dynkin diagrams of A,*

D, E-type. These M-structures and their representations can be described in terms of quiver representations. We investigate in details the multiplications of the $\tilde A_{2 k-1}$-type and integrable matrix ODEs and PDEs generated by them.

Speaker: V.P. Spiridonov (Laboratory of Theor. Physics, JINR, Dubna, Russia)

Title: Elliptic hypergeometric functions in integrable systems and superconformal field theories.

Abstract: *Elliptic hypergeometric functions form a new class of special functions of mathematical physics. We shall briefly survey their appearance in integrable systems in the context of elliptic solutions of the Yang-Baxter equation, solutions of L-A pair equations for a generalized discrete time chain, and finite difference Calogero-Sutherland type models. However, the main attention will be paid to recent supersymmetric interpretation of the elliptic hypergeometric integrals as superconformal (topological) indices of supersymmetric field theories. Connections with the supersymmetric quantum mechanics, Witten index, and the Darboux type transformations will be mentioned in appropriate places.*

Speaker: Wen-Li Yang (Northwest University - Xian, P. R. China)

Title: Complete spectrum of the open spin chain

Abstract: *The multiple reference state structure of the open XXZ chain with non-diagonal boundary terms is studied. The corresponding two sets of Bethe states constitute the complete eigenstates. This talk is based on the papers*

- *W.,-L. Yang, R.,I. Nepomechie and Y.,-Z. Zhang,* "Q-operator and $T-Q$ relation from the fusion hierarchy*", Phys. Lett.* ***B 633*** *(2006), 664-670, e-print:hep-th/0511134.*
- *W.,-L. Yang and Y.,-Z. Zhang, "*On the second reference state and complete eigenstates of the open XXZ chain"*, J. High Energy Phys.* ***JHEP 04*** *(2007), 044 (11 pages), e-print:hep-th/0703222.*
- *W.,-L. Yang and Y.,-Z. Zhang,"* Multiple reference states and complete spectrum of the Z_n Belavin model with open boundaries"*, Nucl. Phys. B* ***789*** *(2008), 591-609, arXiv:0706.0772*
- *W.,-L. Yang and Y.,-Z. Zhang,"* Drinfeld twists of the open XXZ chain with non-diagonal boundary terms"*, Nucl. Phys. B* ***831*** *(2010), 408-428.*